former writers, the author instituted several sets of experiments; some calculated to determine the forces required for dragging bodies of various kinds along a horizontal surface, and others for measuring the angle at which a plane was required to be inclined to the horizon in order to admit of the body sliding down it, attention being paid to the circumstances of pressure, extent of surface, time of previous contact, and velocity of motion.

The following are the principal conclusions which the author deduces from his experiments. The friction of ice rubbing upon ice diminishes with an increase of weight; but without observing any regular law of increase. When dry leather is made to move along a plate of cast iron, the resistance is but little influenced by the extent of surface. With fibrous substances, such as cloth, the friction diminishes by an increase of pressure, but is greatly increased by the surfaces remaining for a certain time in contact; it is greater, *cæteris paribus*, with fine, than with coarse cloths; the resistance is also much increased by an increase of surface. With regard to the friction of different woods against each other, great diversity and irregularity prevail in the results obtained; in general the soft woods give more resistance than the hard woods: thus, yellow deal affords the greatest, and red teak the least friction. The friction of different metals also varies principally according to their respective hardness; the soft metals producing greater friction under similar circumstances than those which are hard. Within the limits of abrasion, however, the amount of friction is nearly the same in all the metals, and may in general be estimated at one sixth of the pressure. The power which unguents have in diminishing friction, varies according to the kind of the fluidity of the particular unguent employed, and to the pressure applied.

The paper is accompanied with drawings of the apparatus used; and the details of the experiments are given at length in a tabular form.

*An Attempt to rectify the Inaccuracy of some Logarithmic Formulæ.* *By* John Thomas Graves, *of the Inner Temple, Esq.* *Communicated by* John Frederick William Herschel, *Esq. V.P.* Read December 18, 1828. [*Phil. Trans.* 1829, *p.* 171.]

The discovery made by Poisson and Poinsot during their recent researches on angular sections, of errors in trigonometrical formulæ usually deemed complete, drew the attention of the author to an analogous incorrectness in logarithmic series. He accordingly proposes in the present paper to exhibit in an amended form two fundamental developements; the principles employed in the establishment of which admit of application in expanding by different methods various similar functions, and tend also to elucidate other parts of the exponential theory.

He then enters into an analytical investigation of the equation $a^x = y$, and exhibits correct developements; first, of $y$ in terms of

$a$ and $x$; and secondly, of $x$ in terms of $a$ and $y$; the corresponding developements hitherto given being incomplete. He considers the principles employed in this inquiry as presenting a solution of many difficulties, and illustrating peculiarities appertaining to the theory of logarithms of negative quantities; and when applied to geometry, as furnishing the means of tracing the form and developing the properties of curves whose equations involve exponential quantities. He also states that by their means various differential and other formulæ usually exhibited in treatises on logarithms may be rendered complete. An appendix is subjoined containing several examples of these applications of his principles. In the course of his investigations, the author endeavours to explain the remarkable anomaly which frequently presents itself to the analyst, of developements, in which, upon substituting a particular value for the variable in each, there is no approximation to numerical identity between the several resulting series, calculated to any number of terms, and the respective functions which they ought to represent. He combats the paradoxical opinion which has been advanced, that equations, which in particular instances are numerically false, are yet analytically true; and explains the difficulty by reverting to the limitations inherent in the hypothesis upon which the developement is founded. He maintains, in opposition to the opinions of Jean Bernouilli and D'Alembert, that the logarithms of negative and positive numbers are not in general the same; and hence infers that negative numbers have occasionally even real logarithms. The chief novelty of his system consists in showing that any assigned quantity, relatively to a given base, has an infinite number of orders of logarithms, and an infinite number of logarithms in each order.

*On the Reflection and Decomposition of Light at the separating Surfaces of Media of the same and of different refractive Powers.* *By* David Brewster, *LL.D. F.R.S. L. & E.* Read February 12, 1829. [*Phil. Trans.* 1829, *p.* 187.]

When white light is incident upon a surface which separates two different media, the portion that is reflected should, according to the Newtonian theory of light, preserve its whiteness, provided the thickness of either of the media exceed the eighty millionth of an inch. But since the dispersive powers of bodies are different, it must follow as a necessary consequence, that reflected light can never under any circumstances retain perfect whiteness, although the modification it experiences is not of sufficient amount to become sensible in ordinary experiments. The author during his investigations of the laws of polarization for light reflected at the separating surface of different media, had occasion to inclose oil of cassia between two prisms of flint glass, and was surprised to find that the light reflected was of a blue colour. The fact was new, but might be readily explained upon the principle that although the refractive density of oil of cassia greatly exceeds that of flint glass for the mean rays, yet the action